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OPTIMIZING BUDGET ALLOCATION FOR ADVERTISING ACROSS VARIOUS MEDIA FOR MAXIMUM AUDIENCE EXPOSURE

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ABSTRACT

Selection of media and budget allocation is a major concern in advertising. It involves choosing the appropriate media that is effective in reaching the target audience of the population in consideration. Generally, predicting an optimal target audience exposure in selection of media is a complex problem. In this study, a Linear Programming technique is used to investigate the budgetary allocation of a Company for effective media selection planning. The problem is formulated using empirical data from the Airtel Company and other media sources. The resulting linear programming model is analyzed using Quantitative Manager for Business version 6.2 and Linear Programming Solver version 7.2.2 which embodies the simplex algorithm method. It was found that out of the three major media categories which give a total of Forty-Five (45) media outlets in the country and a budget of Six Hundred Thousand (N600,000), the results show that the optimal target audience is 541,399,986. The optimum media mix which attracts a significant audience exposure and generates the desired objective value for the advertising campaign include; three (3) Television media outlet, seven (7) print media outlet, and twenty (20) radio media outlet, radio stations respectively for the budget allocation on the major media categories in order to optimize the target audience exposure for the length of the programme.

Keywords: optimization, linear programming, budget allocation, media outlet.

1.0 INTRODUCTION

Media selection planning refers to the strategic process of determining the most suitable channels and platforms for disseminating advertising messages to target audiences. It involves evaluating various media options, such as television, radio, print, digital, and social media, and making informed decisions based on factors such as audience demographics, reach, frequency, cost, and effectiveness. Media selection planning aims to optimize the allocation of advertising budgets to maximize the impact and efficiency of marketing campaigns.

Marketing companies and manufacturers engage in advertising campaigns to inform potential customers about products or services or to remind them of their presence. The goal of the advertiser is to persuade individuals to purchase a product or service they may not have otherwise considered. The advertiser anticipates that their investment in advertising will yield a return or some form of measurable performance for the company. Advertising is not limited to corporations and businesses; individuals, government agencies, institutions, and social organizations also utilize advertising to promote causes or publicize services they offer. In any of these instances, advertisers purchase space or airtime in various media channels, such as television, billboards, radio, or newspapers, to reach their target audiences. They carefully select which media category or combination of categories to utilize in order to effectively convey their advertising message to the desired audience.

This was primarily attributable to the limited number of media categories and vehicles available, coupled with the relatively modest utilization of advertising as a primary promotional tool. Moreover, media selection specialists of the time predominantly relied on subjective models based on expert judgment, alongside simplistic arithmetic models utilizing data centered around circulation and media costs. However, with the increasing prominence of advertising as a pivotal promotional tool and the advent of more sophisticated media data, the landscape of media selection planning underwent a significant transformation, becoming progressively more intricate. For example, as highlighted by David (2020), an approximate sum of 27 billion dollars annually is invested in the United States across the eight major media channels, underscoring the considerable economic significance of media decision-making for marketing entities in the contemporary era.

As a result, media selection planning has become a central focus for the creation of various models within the marketing sector, outpacing other issues in the field, as noted by Kotler (2019). This research, we will focus on the application of specific optimization techniques, particularly the linear programming model, to explore the concept of the media selection planning problem.In the media selection planning process, the challenge is to select from among variousmedia alternatives the "best set". Given that the total budget available is a restraint.Alternatives include not only media, but specific choices within a given medium as well.For a given newspaper, for example, there is the choice of page size, colors and the like.Thus, choices available include all media capable of carrying out an advertisement toachieve the desired results. Each medium is expected to yield a certain number ofexposures per Nigeria Naira. Clearly, the term exposure will be defined in such a way thatexposures among various alternatives can be compared within the media categories.

Linear programming (LP) is a mathematical optimization technique used to determine the best possible outcome in a given situation, subject to specific constraints. It is widely applied in operations research, economics, business and engineering.

1.1 Statement of the Problem

In Nigeria, the utilization of a conceptual model for selection poses a significant challenge, with many companies showing a propensity towards minimal or no conceptual approach in their media campaigns. This issue is often perceived as an insurmountable hurdle within the country (Kwansah-Aidoo, 2013). It is against this backdrop that the adoption of the linear programming conceptual model is advocated to highlight the distinct advantages of employing a conceptual approach. Conceptual models offer the opportunity to optimize media selection planning by considering the impact of each variable in relation to its cost and effectiveness, a facet lacking in the heuristic approach traditionally employed in Nigeria. Consequently, the incorporation of linear programming as a conceptual model becomes imperative.

Linear programming provides a natural framework for examining the media selection challenge. It is employed for situations where there exists a multitude of options for distributing limited resources among competing alternatives to achieve the optimal value of a specified criterion function, while adhering to various constraining conditions. In the context of media selection, the limited resource is typically the media budget, and the available media options, such as Television, Radio, and Print, represent alternative avenues for allocating this budget. The primary constraints in media selection include the magnitude of the media budget and the prescribed minimum and maximum utilization levels for specific media channels. The optimal combination is determined based on effectiveness criteria. The criterion function aims to incorporate factors such as the target audience of the advertiser, the varying levels of prestige associated with different media channels, and the diverse exposure values of the respective units within each channel, thereby influencing the decision-making process of the linear programming model.

1.2 Aim and Objectives of the Study

The Aim of the study is to apply linear programming to optimize the allocation of an advertising budget across various media for maximum audience exposure within identified policies and restrictions. Specifically, the Objective are to, Evaluate the effectiveness of Linear Programming in optimizing AIRTEL Company's advertising budget.Identify the key factors influencing the optimal media mix for AIRTEL Company using Linear Programming.Derive insights from sensitivity analyses to assess the robustness of AIRTEL's media selection planning.

2.0 LITERATURE REVIEW

Optimization or constraint optimization is the process of obtaining the best possible 'result' under the 'circumstance'. The result is measured in terms of an objective which is either to maximize or minimize, whilst the circumstances are defined by a set of equality and/or inequality constraints (Murtagh and Saunders, 2019). Optimization originated from George Dantzig, a member of the United State Air Force who developed the simplex method for solving constraint optimization problems in (2013).

By leveraging LP techniques, this research aims to provide valuable insights into the optimal allocation of advertising budgets for AIRTEL Company, thereby contributing to more effective and efficient media selection planning.: $f(x_1, x_2, \ldots, x_n)$

Subject to: $g_i(x_1, x_2, \ldots, x_n) \le b_i 1 \le i \le p$

$g_i(x_1, x_2, \ldots, x_n) \le b_i p1 + 1 \le i \le k$

$g_i(x_1, x_2, \ldots, x_n) \le b_i k + 1 \le i \le m$

$x_1, x_2, \ldots, x_n \ge 0$

The functions $f(x_1, x_2, \ldots, x_n)$ and $g_i(x_1, x_2, \ldots, x_n) 1 \le i \le M$ is linear. The decisionvariables $x_j = (x_1, x_2, \ldots, x_n)^T$ values which optimize the objective function of the LPmodel above and satisfy all the constraints equations and inequalities are referred to asthe "optimal decision". The term $b_i (1 \le i \le m)$ is called the right-hand-side (RHS) andrepresents the upper and lower limit imposed on the ith constraint function. Theconditions $x_1, x_2, \ldots, x_n \ge 0$ restrict the decision variables to non-negative real numbersonly. Since the objective and constraints function are linear they are precisely defined inthe form:

$f(x_1, x_2, \ldots, x_n)=$

$c_1 x_1 + c_2 x_2 + \ldots c_n x_n = \sum_j^n j = 1 c_j x_j$ (2.1)

$g_i(x_1, x_2, \ldots, x_n) =$

$a_{i1} x_1 + a_{i2} x_2 + \ldots a_{in} x_n = \sum_j^n j = 1 c_{ij} x_{ij}$

(2.2)

The LP in (2.1) and (2.2) can thus be written in the following form:

Optimize: $\sum_{j=1}^{n} c_j x_j$

Subject to: $\sum_{j=1}^{n} a_{ij} x_j \le b_i 1 \le i \le p$

$\sum_{j=1}^{n} a_{ij} x_j = b_i p + i \le k$

$\sum_{j=1}^{m} a_{ij} x_j \ge b_i k + 1 \le i \le m$

$x_j \ge 0 1 \le j \le n$

The quantities a_{ij} and c_j are called Technological and Cost Coefficients respectively.These together with the RHS b_i, constitute the main parameters of the models.

The following is a brief survey on media selection planning problems solved by othermathematical programming techniques not necessarily linear programming. This surveyis intended to provide insights into some of the existing techniques and algorithmsemployed to tackle these problems.

Pagan and Moore (2019) proposed a goal programming (OP) approach to design and implement a media mix plan for a private physician's office. Their study aimed to achieve two primary objectives: (1) maximize the yearly target exposure and (2) minimize both annual and quarterly budgets for the office. To develop the model using this optimization technique, they analyzed the number of ads placed in specific local media groups (radio, newspapers, magazines, church bulletins, and publications), as well as internet listings. Audience exposure and cost data were collected over one month and then extrapolated on a quarterly basis.

3.0 METHODOLOGY

This study employed Linear Programming techniques to analyze the budget allocation of AIRTEL Company for effective media selection planning. The problem was formulated as a linear programming model using available data from the company and various media sources. Mathematical software tools such as Quantitative Manager for Business version 6.2 and Linear Programming Solver version 7.2.2 will be utilized to solve the resulting linear programming model. Additionally, sensitivity analyses will be conduct on selected parameters to assess the robustness of the revised model to slight variations in input parameter values.

The Decision Variables

The decision variables are the various media outlets from which the advertiser mustchoose to develop the advertising campaign. the decision variables aremade up of five (5) from TV, ten (10) from print and thirty (30) from radio media. Thedecision variables are integer variables defined as:

The objective Function

The objective of the problem is to find how to allocate the advertising budget across thevarious media so as to maximize the total target audience exposure for the campaign,whilst operating under the budgetary and policy restrictions presented earlier. Buying aunit space or time in each media is to a certain number of audience exposures. Therefore,audience exposure associated with each media is a parameter of the objective function.Let $(A_{t_i}, A_{Pj}, A_{r_K})$ and let $(x_{t_i}, x_{Pj}, x_{rk})$where A_{t_i}, A_{Pj}, A_{r_K} are respectively row vectors for TV, Print, and Radio audience exposures Xti, Xpj, Xrk representing thedecision variables.

TELEVISION MEDIA

$Z = 447{,}599x_1 + 895{,}199x_2 + 413{,}169x_3 + 516{,}461x_4 + 103{,}292x_5$

PRINT MEDIA

$2{,}000{,}000y_1 + 500{,}000y_2 + 900{,}000y_3 + 750{,}000y_4 + \ldots\ldots\ldots\ldots + 650{,}000y_{10}$

RADIO MEDIA

$65{,}655.84z_1 + 32{,}827.92z_2 + 87{,}541.92z_3 + 21{,}885.28z_4 + \ldots\ldots\ldots + 28{,}279.2z_{30}$

The constraints

The constraints of the problem could broadly be classified into two groups; the first group dealt with the budget allocation to the respective media and their relationship between them. The next group also dealt with the number of adverts available for each media category per the length of the advertising campaign.

The Budget constraints

- **TV Media**

Total Budget for TV media: $\sum_{i=1}^{5} C_{t_i} x_{t_i} \leq 200{,}000$……………………………………………(6)

$780x_1 + 900x_2 + 500x_3 + 427x_4 + 590x_5 \leq 200{,}000$

TV Media budget in relation to Print Media budget: $\sum_{i=1}^{5} C_{t_i} x_{ti} - \sum_{j=1}^{10} C_{Pj} x_{pj} > 0$….(7)

- **Print Media**

Total budget for print media: $\sum_{j=1}^{10} C_{Pj} x_{pj} \leq 150{,}000$……………………………………….(8)

$3928.12y_1 + 970.70y_2 + 1,614.72y_3 + \ldots\ldots + 682.12y_{10} \leq 150,000$

- **Radio Media**

Total budgetfor Radio Media:$\sum_{k=1}^{30} C_{rk}x_{rk} \leq 250,000$……………………………………..(9)

$103z_1 + 68z_2 + 151z_3 + 25.9z_4 + \ldots\ldots\ldots + 42z_{30}$

4.0 Results

Summary of Sensitivity Analysis

Media category &budget	Optimal value	Optimal value up by 10%	Optimal valuedown by 10%
TV Media Group	541,399,986	562,764,255	520,035,717
Difference	0	21,364,269	21,364,269
% Value	0	4%	4%
Print Media Group	541,399,986	551,894,986	530,940,986
Difference	0	10,495,000	10,495,000
%value	0	2%	2%
Radio Media Group	541,399,986	563,680,721	519,119,257
Difference	0	22,280,735	22,280,729
%value	0	4%	4%
Audience exposure value	541,399,986	595,539,974	487,259,993
Difference	0	54,139,988	54,139,993
%value	0	10%	10%
Budget Values	541,399,986	578,780,665	502,271,807
Difference	0	37,380,679	39,128,179
% value	0	7%	7%

The results as in Table 4.8 showed that the recommended media mix for the study includes some of each candidate media which was collectively given as three (3) from Television, seven (7) from print, and twenty (20) from radio media outlets. These recommended media mix identified would generate an optimal value of 541,399,986 target audience exposure as the objective that the company was intended to achieve within the period of the advertising campaign programme.

Also, sensitivity analysis test was conducted on some of the parameter values and their resulting models were compared with the original LP problem. The comparison of the results as presented in table 4.8 shows that they all proposed some media outlets within each media category.

Next, the right - hand - side (RHS) of the budgetary constraints values was varied at 10% level, so as to gauge the effect on the objective function value. In the first scenario, each of the RHS of the budgetary constraints values was decreased by 10% and this led to a corresponding reduction in the optimal value to 502,271,807 exposures per the length of the programme. Conversely, an increased in the RHS of the budgetary constraint values by the same percentage increased the optimal value to 578,780,665 exposures.

An interesting observation that was made in the RHS budgetary values variation was that new variables either entered or left the basis which lead to the creation of new media mix for the advertising campaign programme. Again, some of the basis variables increased or decreased in solution values which lead to a subsequent effect on the optimum value.

Further, table 4.8 shows that increasing each of the media category exposure values one at a time by 10% would lead to a corresponding increase in the objective function value, likewise decreasing each of the media category exposure values by the same percentage would lead to a reduction in the objective function value with their respective percentages. The study observed that as all the audience exposures values were varied by 10% upward, the objective function value increased significantly to 595,539,974 exposures and this corresponds to about 54,139,988 increments in the optimal value.

Similarly, as all the audience exposures were varied downward by 10%, the objective function value decreased drastically to 487,259,993 exposures and this corresponds to about 54,139,993 lost in the optimal value. The solution values of the decision variables were the same as compared with the original LP problem. Moreover, all the audience exposure and the RHS budgetary constraints values were subjected to varied up and down by 10%, the results of the revised model shows that the objective function value increased to 552,498,977 exposures and this corresponds to an increment of about 11,098,991 exposures in the optimal value. On the contrary, as all the audience exposures and the RHS budgetary constraints values were varied down and up by the same percentage, the resulting objective function value decreased to 520,902,609 which corresponds to a reduction of about 20,497,377 exposures in the optimal value.

5.0 Conclusion

In the process of formulating the media selection planning problem for the AIRTEL Company, we attempted to simulate the actual operation as closely as possible in order to produce a realistic model that would be used for the purchasing of advertising space or time in the various media outlets considered in the research. The output of the model was ascertained by using LP programming techniques in conjunction with computer software packages. It was found that out of the three major media categories which give a total of Forty-Five (45) media outlets in the country and advertising budget of Six Hundred Thousand Nigeria Nairas (N600,000), the optimal target audience exposure was 541,399,986. The optimum media mix which generated the desire objective value for the advertising campaign include; three (3) Television media outlet, seven (7) newspapers within the print media outlet, and twenty (20) FM stations within the radio media outlet respectively. Thus, Linear programming techniques has aided us to come out with effective media selection planning for the AIRTEL Company.

By conducting sensitivity analysis test on the model for various parameters option, the AIRTEL company can pre - assess the effect of a given parameters on the overall optimal value. The sensitivity analysis of the model shows that some media outlets such as Nigeria Business, Junior Graphic, Nigeria Sports, Daily Trust for the print media group, Joy FM, Happy FM, Peace FM, for the radio media group, and AFRICA INDEPENDENT TELEVISION (AIT), Viasatl, CHANNELS TELEVISION for the' television media group always remained in the basis regardless of any variation in the model. The presence of these media outlets in the basis implied that they had a tremendous impact on the optimal value.

Moreover, the availability of sufficient computer software such as QM and LP Solver had facilitated the generation of the output of the model easily and within a very short period of time. In addition, the proposed model can serve as a useful tool not only for marketing companies but also media practitioners, stakeholders, institutions, and even individuals for negotiation cost of adverts in the various media outlets in the country.

5.1 Recommendations

One of the most successful and important application of linear programming to solving business problems has been in the area of media selection planning. From the conclusions, it was realized that using quantitative methods to identify the best media mix for advertising companies has helped them to yield the desire audience exposure which will in turn bring about a turn over to the company.

It is therefore recommended that the AIRTEL company should focus on advertising on some of each candidate media outlets; three (3) television, seven (7) newspapers, and twenty (20) radio stations respectively for the budget allocation on the major media categories in order to optimize the target audience exposure for the length of the programme.

Also, I recommended that apart from identifying the best media mix for the advertising campaign programme for just one of the telecommunication company in the country, advertising institutions and other marketing companies should adapt scientific and mathematical approach in most of their advertising campaign programme.In addition, it is recommended that the telecommunication company should adapt this model in the allocation of the advertising budget set for media selection planning problem. Further, marketing companies should be educated to employ quantitative method to find an appropriate quantitative model to assist them in the allocation of advertising budget more efficiently across the various media categories.

In this problem only a single criterion was optimized which is the audience exposure to reach by the company. In any advertisement promotional campaign, management would expect that the advertising budget which is the fixed cost be as small as possible.Therefore, the fixed cost could instead be minimized whilst the audience exposure is maximized in that perspective. Hence, it suffices that a bi - criterion of the media selection planning problem involving these factors as objective to be optimized simultaneously and this could provide a useful insight into the subject area. These lines of investigation would be the subject of future work.

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